Go straight to the integral power rule

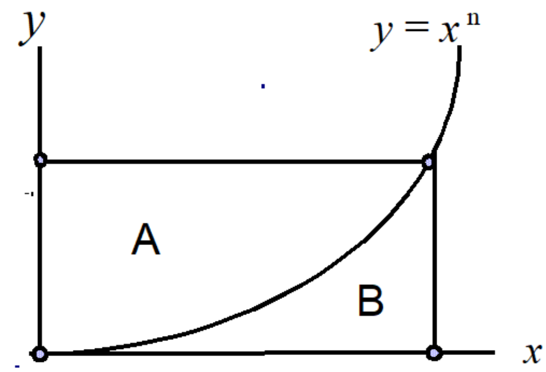
This is a pre-calculus taster designed as an easy way in for students, many of whom are nervous of this topic. I think area is more familiar to students than gradient, so I've used the graph for area 'under' the curve and added a line to show area 'above' the curve. Going for the integral rule first makes them more accepting of later theory.

Using the mid-ordinate rule, calculate the value of area B for $x = 10$ and n = 2. Check that using 10 vertical strips, $\mathrm{B} = 335.05$

$y = 100 \therefore xy = \mathrm{A} + \mathrm{B} = 1000$. Calculate A and check that $\dfrac{\mathrm{A}}{\mathrm{B}} = 2.00256$

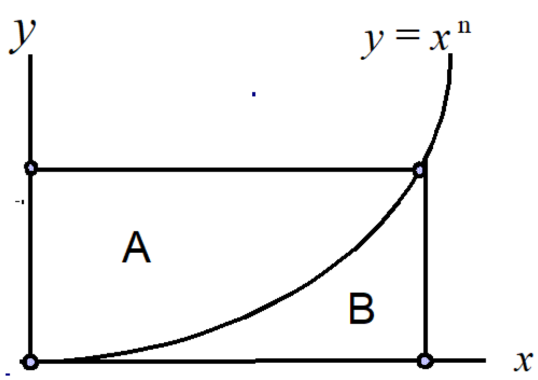
Check that further calculations for n = 3, 4, 5 show that $\dfrac{\mathrm{A}}{\mathrm{B}} \approx \mathrm{n}$

Note that the value of $\dfrac{\mathrm{A}}{\mathrm{B}}$ is more accurate when more strips are used and with an infinite number of strips, we can say that $\dfrac{\mathrm{A}}{\mathrm{B}} = \mathrm{n}$



Go to integral power rule: $\mathrm{A} + \mathrm{B} = xy = x^{\mathrm{n}+1}$ and $\dfrac{\mathrm{A}}{\mathrm{B}} = \mathrm{n} \;\therefore \mathrm{A} = \mathrm{Bn} \;\therefore \mathrm{Bn} + \mathrm{B} = x^{\mathrm{n}+1} \therefore \mathrm{B}(\mathrm{n}+1) = x^{\mathrm{n}+1} \therefore \mathrm{B} = \dfrac{x^{\mathrm{n}+1}}{\mathrm{n}+1}$

Above is a look at what can be done by students with a little encouragement. Below is a simple proof of $\dfrac{\mathrm{A}}{\mathrm{B}} = \mathrm{n}$.



Given that $\int x^{\mathrm{n}} dx = \dfrac{x^{\mathrm{n}+1}}{\mathrm{n}+1}$ (constant of integration omitted),

then in the diagram on the left, $\mathrm{B} = \dfrac{x^{\mathrm{n}+1}}{\mathrm{n}+1}$ Equation 1

Also, $\mathrm{A} + \mathrm{B} = xy \therefore \mathrm{A} + \mathrm{B} = x^{\mathrm{n}+1} \therefore \mathrm{A} = x^{\mathrm{n}+1} - \mathrm{B} \;\therefore \mathrm{A} = x^{\mathrm{n}+1} - \dfrac{x^{\mathrm{n}+1}}{\mathrm{n}+1}$

$\therefore \mathrm{A}(\mathrm{n}+1) = x^{\mathrm{n}+1}(\mathrm{n}+1) - x^{\mathrm{n}+1} \therefore \mathrm{A} = \mathrm{n}\dfrac{x^{\mathrm{n}+1}}{\mathrm{n}+1}$ Equation 2

Comparing equations 1 and 2, we see that $\dfrac{\mathrm{A}}{\mathrm{B}} = \mathrm{n}$

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